Linhas de Transmissão - Parte 5 ~~$$$~~ Chapter 11

Example 11.3

Considere uma L.T. c/ as seguintes características:

freq. operação : $\omega = 10^6$ rad/s
constante de atenuação : $\alpha = 8$dB/m
" " fase : $\beta = 1$ rad/m
impedância característica : $Z_0 = 60 + j40$ Ω
comprimento : $l = 2$ m
fonte : $V_g = 10\angle 0°$
impedância da fonte : $Z_g = 40$ Ω
" carga : $Z_L = 20 + j50$ Ω

a) Impedância de entrada, Z_{in}
b) corrente ~~ao longo da linha~~ em $z=0$.
c) " no meio da linha.

a) $1\ Np = 8.686\ dB$ $\qquad 1\ Np \equiv 20/\ln(10) = 8{,}685889638\ dB$

$1\ dB \equiv 0.115129255\ Np$

$$\alpha = 8\frac{dB}{m} \cdot \frac{0.115129255\ Np}{dB}$$

$$\boxed{\alpha = 0{,}921\ Np/m}$$ convenção p/ $\alpha \Rightarrow Np/m$

$$\gamma = \alpha + j\beta = 0{,}921 + j1 \quad m^{-1}$$

$$\gamma l = (0{,}921 + j1)\cdot 2 = 1{,}84 + j2$$

$$\tanh(a+b) = \frac{\tanh(a) + \tanh(b)}{1 + \tanh(a)\cdot\tanh(b)}$$
$$= \frac{\tanh(1{,}84) + j\tan(2)}{1 + \tanh(1{,}84)\cdot j\tan(2)}$$

$$\tan(1{,}84 + j2) = 1.0327607 - j0{,}0394534 \quad *$$

Assim, a impedância Zin :

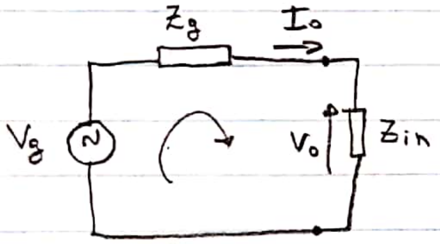
$$Z_{in} = Z_0 \cdot \frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)}$$

$$Z_{in} = (60+j40) \cdot \left[\frac{20+j50 + (60+j40)\cdot(1{,}0327607 - j\,0{,}0394534)}{60+j40 + (20+j50)\cdot(1{,}0327607 - j\,0{,}0394534)}\right]$$

$$\boxed{Z_{in} = 60{,}25 + j\,38{,}79 \;\; \Omega} \; *$$

b) Corrente de entrada em $z=0$

Utilizando o circuito equivalente



$$V_g = I_0 Z_g + V_0$$

$$V_0 = I_0 Z_{in}$$

$$V_g = I_0 (Z_g + Z_{in})$$

$$I_0 = \frac{V_g}{Z_g + Z_{in}}$$

$$= \frac{10\angle 0^\circ}{40 + 60{,}25 + j\,38{,}79} = \frac{10\angle 0^\circ}{100{,}25 + j\,38{,}79} = \frac{10\angle 0^\circ}{107{,}493\angle 21{,}153^\circ}$$

$$I_0 = 0{,}09303 \angle -21{,}153^\circ$$

$$\boxed{I_0 = 93{,}03 \angle -21{,}153^\circ \;\; mA} \; *$$

c) Encontre a corrente em qualquer ponto.

$$I_s(z) = \frac{1}{Z_0}\left[V_{s0}^{+} e^{-\gamma z} - V_{s0}^{-} e^{\gamma z}\right]$$ da equação 2

precisamos encontrar V_{s0}^{+} e V_{s0}^{-}

$$V_s(z=0) = V_0 = V_{s0}^{+} + V_{s0}^{-}$$

$$I_s(z=0) = I_0 = \frac{V_{s0}^{+}}{Z_0} - \frac{V_{s0}^{-}}{Z_0}$$

$$V_{s0}^{+} = V_0 - V_{s0}^{-}$$

$$V_{s0}^{-} = V_{s0}^{+} - I_0 Z_0$$

$$V_{s0}^{+} = V_0 - V_{s0}^{+} + I_0 Z_0$$

$$\boxed{V_{s0}^{+} = \frac{1}{2}\left(V_0 + I_0 Z_0\right)}$$

$$V_{s0}^{-} = \frac{1}{2}\left(V_0 + I_0 Z_0\right) - I_0 Z_0$$

$$V_{s0}^{-} = \frac{1}{2} V_0 + \frac{1}{2} I_0 Z_0 - I_0 Z_0$$

$$\boxed{V_{s0}^{-} = \frac{1}{2}\left(V_0 - I_0 Z_0\right)}$$

Scanned with CamScanner

Do circuito equivalente do item (b)

$V_0 = I_0 \, Z_{in}$

$V_0 = 93{,}03 \angle -21{,}153° \cdot (60{,}25 + j\,38{,}79)$

$V_0 = 93{,}03 \angle -21{,}153° \cdot 71.657 \angle 32{,}774°$

$\boxed{V_0 = 6{,}666 \angle 11{,}621°}$

Sabemos também q/ :

$V_s(z) = V_{s0}^{+} e^{-\gamma z} + V_{s0}^{-} e^{\gamma z}$

A tensão e corrente V_0 e I_0 correspondem a $V_s(z=0)$ e $I_s(z=0)$, assim basta substituirmos

$V_{s0}^{+} = \frac{1}{2}(V_0 + Z_0 I_0)$ ver ao lado

$V_{s0}^{-} = \frac{1}{2}(V_0 - Z_0 I_0)$

$V_{s0}^{+} = \frac{1}{2}\left(6{,}666 \angle 11{,}621° + (60 + j40) \cdot 93{,}03 \angle -21{,}153° \text{ mA}\right)$

$\boxed{V_{s0}^{+} = 6{,}687 \angle 12{,}08°}$ *

$V_{s0}^{-} = \frac{1}{2}\left(6{,}666 \angle 11{,}621° - (60 + j40) \cdot 93{,}03 \angle -21{,}153° \text{ mA}\right)$

$\boxed{V_{s0}^{-} = 0{,}0518 \angle 260°}$ *

No meio da linha $\Rightarrow z = \ell/2 \Rightarrow \gamma z = 0{,}921 + j1$

$I_s(z = \ell/2) = \frac{V_{s0}^{+}}{Z_0} e^{-\gamma z} - \frac{V_{s0}^{-}}{Z_0} e^{\gamma z}$

$$I_s(z=\ell/2) = \frac{\left(6,687\, e^{j12,08°}\right) \cdot e^{(-0,921-j1)}}{60+j40} - \frac{\left(0,0518\, e^{j260°}\right) e^{0,921+j1}}{60+j40}$$

$$I_s(z=\ell/2) = \frac{6,687 e^{j12,08°} \cdot e^{-0,921} \cdot e^{-j57,3°}}{72,1\, e^{j33,69°}} - \frac{0,0518\, e^{j260°} \cdot e^{0,921} \cdot e^{j57,3°}}{72,1\, e^{33,69°}}$$

$$I_s(z=\ell/2) = 0,0369\, e^{j78,91°} - 0,001805\, e^{j283,61°}$$

$$= 6,673 - j\,34,456 \text{ mA}$$

$$\boxed{I_s(z=\ell/2) = 35,10 \angle 281° \text{ mA}}$$

*

Exercício: Sugestão p/ os alunos resolverem

(1) LT de 40m, $V_g = 15\angle 0°$ V_{rms}, $Z_0 = 30 + j60\,\Omega$
$V_L = 5\angle -48°$ V_{rms}.
Supondo Linha casada com a carga, calcule

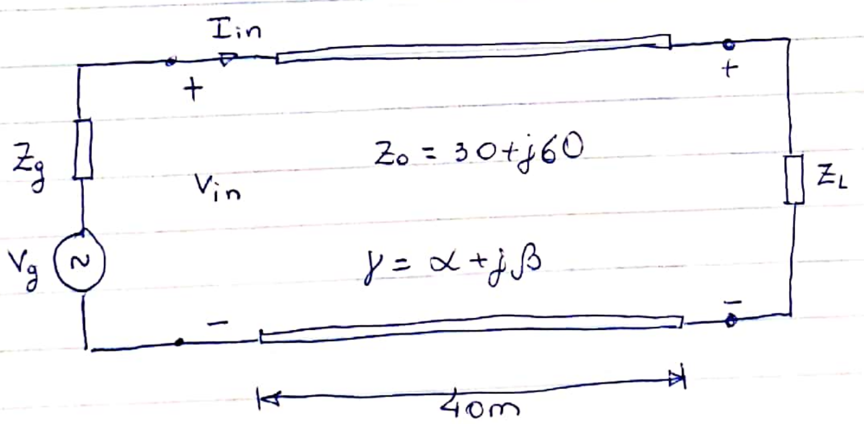
\# $\boxed{\text{practice exercise } 11.3}$

a) Z_{in}
b) corrente de entrada I_n e tensão V_{in}
c) constante de prop. γ

Resposta:
(a) $30 + j60\,\Omega$

(b) $0{,}112\angle -63{,}43°$ A $\quad 7{,}5\angle 0°$ V_{rms}

(c) $0{,}0101 + j\,0{,}2094$ /m



Practice ② L.T. operando em $f = 500MHz$ p/ alunos
Exercise
11.1
$Z_0 = 80\,\Omega$
$\alpha = 0{,}04\ Np/m$
$\beta = 1{,}5\ rad/m$

Encontre: R, L, G, e C

$R = 3{,}2\,\Omega/m$ $G = 5\times10^{-4}\ S/m$
$L = 38.2\ nH/m$ $C = 5{,}97\ pF/m$

Example
11.2 ③ Uma L.T sem distorção apresenta:

$Z_0 = 60\,\Omega$ $u = 0{,}6\,c$ $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$
$\alpha = 20\ mNp/m$

Encontre:

R, L, G, C. e λ p/ $f = 100MHz$

$RC = GL$ ou $G = \dfrac{RC}{L}$

portanto:

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\alpha = \sqrt{RG} = \sqrt{R.\frac{RC}{L}} = R\sqrt{\frac{C}{L}} = \frac{R}{Z_0}$$

Assim, $\boxed{R = \alpha Z_0}$ $R = (20\times10^{-3}).(60) = 1{,}2\,\Omega/m$ *

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

Utilizando:

$$\frac{Z_0}{u} = \sqrt{\frac{L}{C}}\,.\,\sqrt{LC} = L$$

$$L = \frac{Z_0}{u} = 333\ nH/m$$ *

como:

$$G = \frac{\alpha^2}{R} = \frac{400 \times 10^{-6}}{1,2} = 333 \mu S/m \ *$$

fazendo

$$Z_0 \cdot u = \sqrt{\frac{L}{C}} \cdot \frac{1}{\sqrt{LC}}$$

$$Z_0 u = \frac{1}{C}$$

$$C = \frac{1}{Z_0 . u} = \frac{1}{0,6 \cdot (3 \times 10^8) 60} = 92,59 \ pF/m \ *$$

$$\lambda = \frac{u}{f} = \frac{0,6 \ (3 \times 10^8)}{10^8} = 1,8 \ m \ *$$